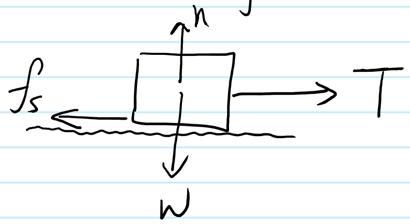
Lecture 3 : Newton's Laws II

Friction — contact friction, microscopic : due to intermolecular forces e.g. Van der Waals force

— fluid Resistance (air drag)

Contact friction

① Static friction.



- Before it moves, $\vec{a} = \vec{0}$
  $\Rightarrow f_s = T \quad \because$ 1st Law.

- When T increases, f_s increases, too.

- Until the block starts to move,
  $\vec{a} \neq \vec{0} \quad \Rightarrow \quad T > f_s$

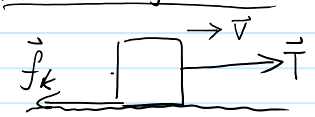
  f_s saturates at a maximal value : $f_{s,max}$.

- $f_{smax} \propto n \Rightarrow \boxed{f_{smax} = \mu_s n}$

  coefficient of static friction.

- properties of f_s :
  - varies with other forces
  - saturates at $f_{smax} = \mu_s n$.
  - direction opp. to resultant of other $\vec{F}$. & tangential to the surface

② Kinetic friction.



Direction : $\vec{f}_k \perp \vec{n}$, tangential to surface & opp. to $\vec{v}$

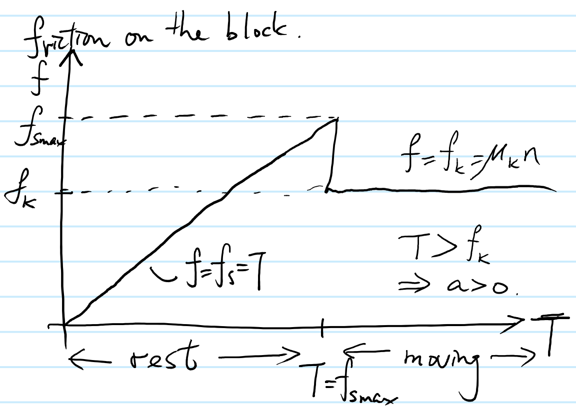
Magnitude : $f_k = \mu_k n$ (always)

coefficient of kinetic friction.

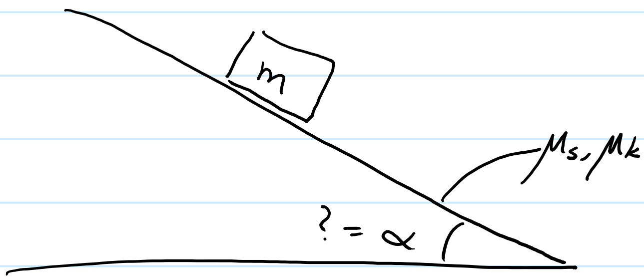
Experiment shows

$\mu_k < \mu_s$

i.e., easier to keep sth in motion than starting a motion.



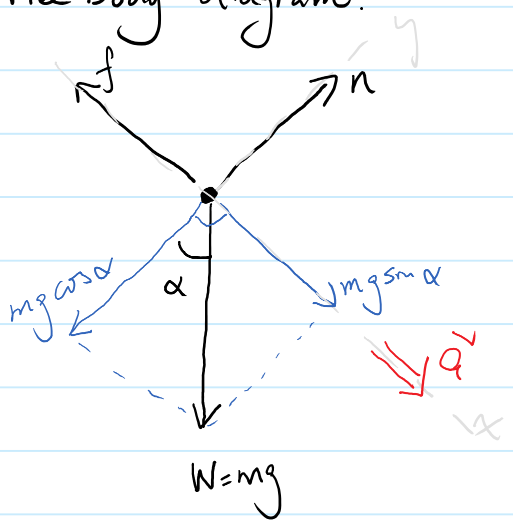
Incline plane



Q: ① at what α, will the block start to slide?

② once it slides at that angle α, what is the acceleration?

<u>Step 1</u> Free body diagram.



<u>Step 2</u> define coordinate, decompose vectors, indicate $\vec{a}$

<u>Step 3</u> write $\Sigma \vec{F} = m\vec{a}$ along x- & y- axis.

$$x: \quad mg\sin\alpha - f = ma \qquad \text{——— ①}$$

$$y \quad n - mg\cos\alpha = 0 \qquad \text{——— ②}$$

<u>Step 4.</u> Solve.

Right before it slides $\vec{a} = \vec{0}$ & $f = f_{smax} = \mu_s n$

$\Rightarrow$ from Eq. ②, $n = mg\cos\alpha$

from Eq ①: $mg\sin\alpha - \mu_s n = ma = 0$

$$\Rightarrow \quad mg\sin\alpha - \mu_s mg\cos\alpha = 0$$

$$\Rightarrow \quad \tan\alpha = \mu_s \quad \Rightarrow \quad \underline{\underline{\alpha = \arctan(\mu_s)}}$$

Q. ② once it moves, friction becomes kinetic friction.

$$f_s \rightarrow f_k = \mu_k n$$

at $\alpha = \arctan(\mu_s)$, $mg\sin\alpha - f_k = ma$

$$\Rightarrow \quad mg\sin\alpha - \mu_k mg\cos\alpha = ma$$

$$\Rightarrow \quad a = g\cdot(\sin\alpha - \mu_k\cos\alpha)$$

$$= g\cos\alpha(\tan\alpha - \mu_k)$$

$\because \cos\alpha = \sqrt{\dfrac{1}{1+\tan^2\alpha}}$

$= \sqrt{\dfrac{1}{1+\mu_s^2}}$

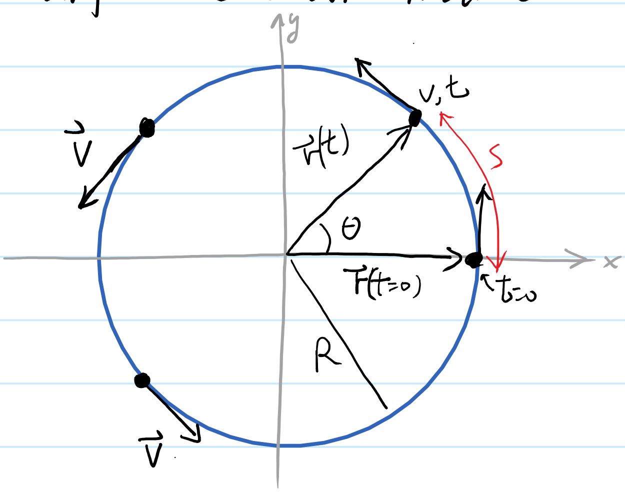
$$\boxed{a = g\frac{\mu_s - \mu_k}{\sqrt{1+\mu_s^2}}} > 0 \quad \because \mu_s > \mu_k.$$

skipped in class.

note: $a = \underbrace{g\sin\alpha}_{\text{acc. on incline without friction}} - \underbrace{\mu_k g\cos\alpha}_{\text{acc. is reduced by friction.}}$

Centripetal acceleration   kinematics in 2D

uniform circular motion



Since $R\theta = s$ & $s = vt$

$$\Rightarrow \quad \theta = \theta(t) = \frac{v}{R} t \equiv \omega t$$

def: $\omega = \frac{v}{R}$. angular velocity (rad/sec.)

position vector: $\vec{r}(t) = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$

$$\vec{r}(t) = R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}$$

Velocity vector: $\vec{V}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt} R\cos\omega t\,\hat{i} + \frac{d}{dt} R\sin\omega t\,\hat{j}$

$$= -\omega R\sin\omega t\,\hat{i} + \omega R\cos\omega t\,\hat{j}$$

acceleration: $\vec{a}(t) = \frac{d\vec{v}}{dt} = -\omega^2 R\cos\omega t\,\hat{i} - \omega^2 R\sin\omega t\,\hat{j}$

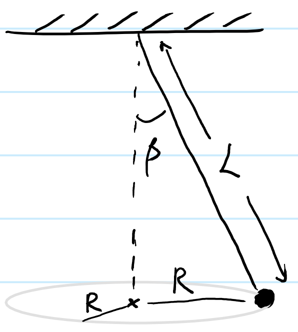
$$\vec{a}(t) = -\omega^2\,\vec{r}(t)$$

opp. to direction of $\vec{r}$. i.e. always pointing at the center.

$$a = |\vec{a}| = \omega^2 |\vec{r}| = \omega^2 R = \frac{V^2}{R}$$

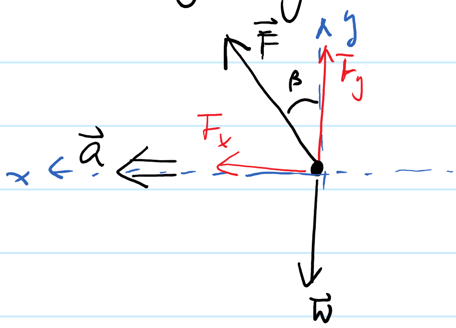
In uniform circular motion: $\begin{cases} a_{rad} = a_{\perp} = \dfrac{V^2}{R} \\ a_{tan} = a_{\parallel} = 0 \end{cases}$

Conical pedulum : Find period T, time to complete one cycle.



$R = L \sin\beta$

Free body diagram:



Newton's Law:

$x: \quad F_x = m a_c \Rightarrow F \sin\beta = m \frac{V^2}{R} \quad — ①$

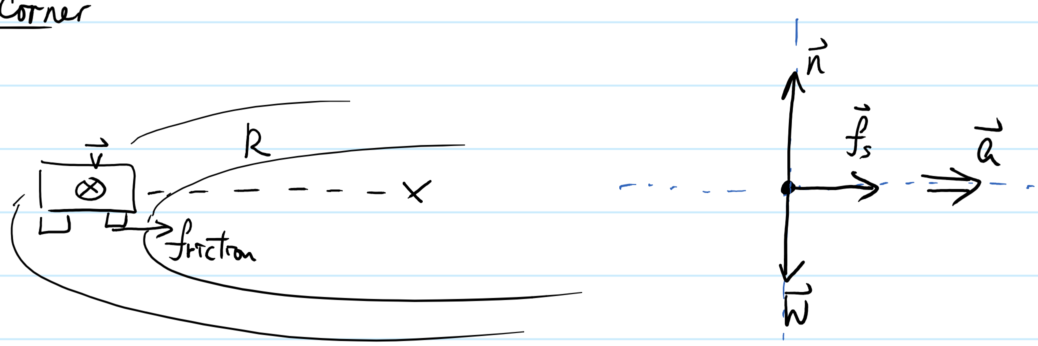
$y: \quad F_y - mg = 0 \Rightarrow F \cos\beta = mg \quad — ②$

①/② gives : $\tan\beta = \frac{V^2}{gR}$ $\qquad \frac{V}{R} = \sqrt{\frac{g \tan\beta}{R}}$

$$\text{Period} = T = \frac{\text{distance}}{\text{speed}} = \frac{2\pi R}{V}$$

$$T = 2\pi \sqrt{\frac{R}{g \tan\beta}} = 2\pi \sqrt{\frac{L\cos\beta}{g}}$$

Car turning corner



when the car's wheels are rolling without slipping, no relative motion between the tires and the road surface occurs. $\Rightarrow$ static friction

Applying Newton's 2nd Law:

$$\begin{cases} f_s = m a_c = m\dfrac{V^2}{R} \\ n = mg \end{cases}$$

as V increases, the required friction increases until the friction reaches its maximum.

$$f_s = f_{s\,max} = \mu_s n = \mu_s mg \quad \text{at} \quad V = V_{max}.$$

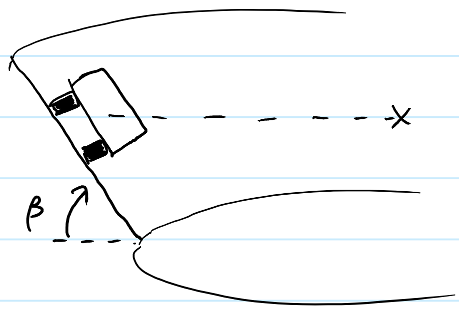
$$\Rightarrow \quad \mu_s mg = \frac{m V_{max}^2}{R}$$

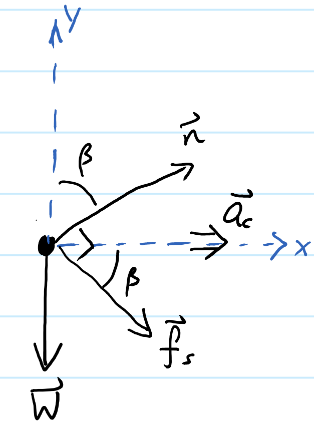
$$\Rightarrow \quad V_{max} = \sqrt{\mu_s g R}$$

The maximum speed to turn a corner depends on μ_s.

How to increase V_{max}?

Banked Curve





Both $\vec{f}$ and $\vec{n}$ provide force to create the centipetal acceleration.

$$x: \quad n\sin\beta + f_s\cos\beta = m a_c \quad \text{— ①}$$

$$y: \quad n\cos\beta - f_s\sin\beta - mg = 0 \quad \text{— ②}$$

① × $\sin\beta$ + ② × $\cos\beta$

$$\Rightarrow n\sin^2\beta + f_s \cancel{\cos\beta\sin\beta} + n\cos^2\beta - f_s\cancel{\cos\beta\sin\beta} - mg\cos\beta = m a_c \sin\beta.$$

$$\Rightarrow n = m(a_c\sin\beta + g\cos\beta)$$

$$f_s = m(a_c\cos\beta - g\sin\beta)$$

where $a_c = \frac{v^2}{R}$

when $f_s = f_{s,max} = \mu_s n$

$$\Rightarrow m\left(\frac{V_{max}^2}{R}\cos\beta - g\sin\beta\right) = \mu_s m\left(\frac{V_{max}^2}{R}\sin\beta + g\cos\beta\right)$$

$$\Rightarrow V_{max} = \sqrt{\left(\frac{\tan\beta + \mu_s}{1-\mu_s\tan\beta}\right) gR} \geq \sqrt{\mu_s g R} \quad \text{for } 0 \leq \beta < \frac{\pi}{2}.$$

When the speed is too slow, the car tends to slide down along the incline. In that case, the friction points up along the incline to help holding the car.

What is the lowest speed of the car without sliding down?

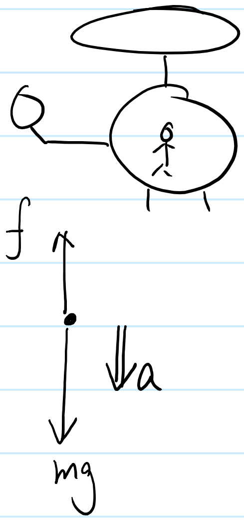
Fluid Resistance

Fluid resistance depends on speed.

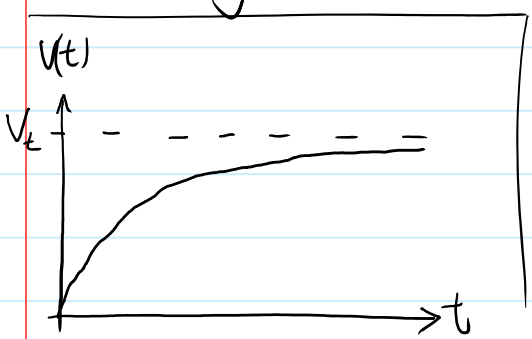
e.g. at high speed (air resistance) $f \propto v^2$ or $f = Dv^2$, $\vec{f} = -Dv^2\hat{v}$

D depends on $\begin{cases} \text{shape}, \\ \rho_{medium}, \text{ etc.} \end{cases}$

sky diving.

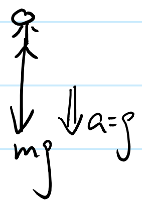


$F_{net} = mg - f = ma$



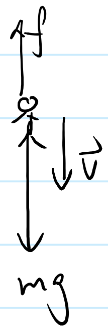
Just jump.

$t=0$



$v=0 \Rightarrow f=0$, $a=g$

t_1



$v>0$, $f>0 \Rightarrow a = g - \frac{f}{m} < g$

a decreases as v still increases.

t_2

now your v gives a f which is as large as mg

$ma = mg - f = 0 \Rightarrow a = 0$

No acceleration any more from this point on.

It has reached the terminal velocity V_t.

What is the value of V_t?

at $v = V_t$, $a = 0$

$\Rightarrow mg = DV_t^2$

$$V_t = \sqrt{\frac{mg}{D}} \propto \sqrt{m}$$

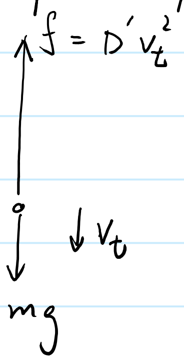
if $m\uparrow$, $V_t\uparrow$ Aristotle: heavier object falls faster.

What if you open a parachute?

$$D \longrightarrow D' > D \quad \Rightarrow \quad V_t \longrightarrow V_t' < V_t$$

safer

When you open a parachute:



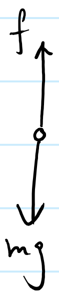
$$f = D'v_t^2 > Dv_t^2 = mg$$

$\Rightarrow \quad f' > mg$

$\Rightarrow \quad \vec{a}$ points upward

$\Rightarrow$ slow down

Until,

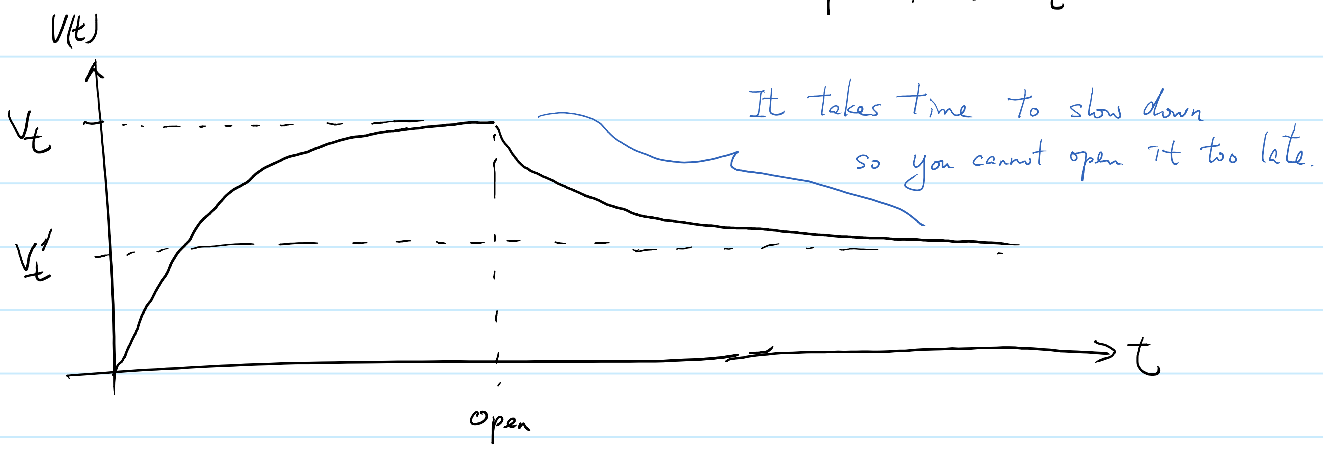


$f = mg$ again.

$$f = D'V_t' = mg \qquad V_t' < V_t$$

$\Rightarrow \quad \vec{a} = 0$

$\Rightarrow$ maintain the same speed at V_t'



projectile